LETTERS TO THE EDITOR

To the Editor:

In a recent article entitled "Dynamic Liquid Holdup in Two-Phase Down Flow in Packed Beds: Air-Silicone Oil System", Clements and Schmidt (1980) determined liquid holdup in the bed by cutting off the liquid flow and collecting the liquid as it drained out. The liquid was collected in timed fraction for a period of thirty minutes. The timed fraction data were plotted in their Fig. 1 and the initial volume of liquid was obtained by extra polation to zero time. The abscissa of Fig. 1 is time t_1 and that of the ordinate is $1/V \times 10^3$ where V = volume of liquid in bed. While the unit of time is given in minutes, there is no mention of the unit of volume of liquid. If V is intended to be the volume fraction referred to the bed volume, there is a problem that the extrapolation of the line in Fig. 1 gives the value of V_0 exceeding 100!

The whole exercise of plotting timed data is not clear. Obviously the authors have made certain assumptions connecting V with the volume drained out from time to time. Furthermore the differntial equation assumed by the authors should have a constant with a negative sign (−k) as the volume of liquid in the bed decreases with the time of drainage.

The authors appear to have neglected the "end corrections" which may, however, be determined by the methods of Reiss (1967) and Charpentier et al. (1975).

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LITERATURE CITED

1. Clements, L. D. and Schmidt, B. C., *AIChE J.*, 36, 317, 1980.
2. Reiss, L. P., Ind. Eng. Chem. Process Des. Dev., 6, 486, 1967.
3. Charpentier, J.C. and Favier, M., *AIChE J.*, 21, 1213, 1975.

To the Editor:

The paper "Revision of Kynch Sedimentation Theory" [*AIChE J.*, 27, 823 (1981)] by Tiller, F. M. makes a substantial contribution to thickening theory. It discovers and demonstrates a functional relationship between H(t), L(t), and u(ϕ), (Tiller notation). Thus from the fall of the top of the suspension in a batch test, and the rise of the suspension-sediment interface, the relation between settling rate u and suspension concentration ϕ can be determined for a significant part of the domain of the relationship. There is, however, an error in equation (29). The term H_1 should be H_0.

Equation (29) results from integration of equation (28):

$$\frac{d\phi_{S2}}{\phi_{S2}} = \frac{d(H_{12} - L_1)}{H_{12} - L_{12}} - \frac{dH_2/dt_2 - dL_1/dt_1}{H_{12} - L_{12}} \qquad (28)$$

For the integration, all the terms are variables, and the integration takes place along the L curve from $t_1 = 0$ to some limit value of t_1, which will be here to designated t^* to distinguish this specified value of t_1 from the symbol t_1 taken as a variable. H_{12} and L_1 are also variables whose values depend on that of t_1. $(H_{12} - L_1)$ relates to the concentration ϕ_{S2} existing just above the compression discontinuity at any time t_1, and is found by making a Tiller construction at whatever value t_1 has at the moment. A Tiller construction made at $t_1 = 0$ is identical to a Kynch construction, and the value of H_{12} is then identical to that of H_z (Tiller Figure 3).

Note that Tiller's construction corresponds to a Kynch construction in a coordinate system whose origin lies at (t_1, L_1).

Integration of (28) proceeds as follows:

$$-dln\phi_{S2} = dln(H_{12} - L_1) - \frac{dH_2/dt_2 - dL_1/dt_t}{H_{12} - L_1}$$

$$[-ln\phi_{S2} \begin{matrix} (t_1 = t^*) \\ (t_1 = 0) \end{matrix} = [ln(H_{12} - L_1)] \begin{matrix} (t_1 = t^*) \\ (t_1 = 0) \end{matrix} - \int_{(t_1=0)}^{(t_1=t^*)} \left(\frac{dH_2/dt_2 - dL_1/dt_1}{H_{12} - L_1} \right) dt_1$$

We do not agree with Tiller's assertion that only the characteristic for ϕ_{So} arises from the origin, and thus that the concentration ϕ_{S2} just above the compression discontinuity at $t_1 = 0$ will necessarily be ϕ_{So}, (this will be treated in forthcoming papers). Fortunately, it makes no difference in this instance. Let ϕ_z denote whatever concentration exists just above the compression discontinuity at $t_1 = 0$, and H_z the value for H_{12} obtained by a Tiller construction at $t_1 = 0$. Then:

$$\phi_{S_2^*} = \left(\frac{\phi_z H_z}{H_{12}^* - L_1^*} \right) e \times \int_0^{t^*} \left[\frac{dH_2/dt_2 - dL_1/dt_1}{H_{12} - L_1} \right] dt_1$$

Where "∗" denotes values of the variables at $t_1 = t^*$

However, from Kynch theory, it follows that for any characteristic arising from the origin, $\phi_z H_z = \phi_{So} H_o$, so:

$$\phi_{S_2^*} = \left[\frac{\phi_{So} H_o}{H_{12}^* - L_1^*} \right] e \times \int_0^{t^*} \left[\frac{dH_{12}/dt_2 - dL_1/dt_1}{H_{12} - L_1} \right] dt_1$$

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To the editor:

We would like to rectify an incorrect statement made in a recent note by Tardos and Pfeffer (1980) regarding the equation for gravitational deposition efficiency of small particles in packed beds given by Rajagopalan and Tien (1976). In addition, we would like to modify the expression for the diffusional deposition efficiency used by Rajagopalan and Tien (1976) so as to make it consistent with the definition of the gravitational efficiency discussed below.

The approximate equation for gravitational collection efficiency in a packed bed modeled by a Happel cell as given in Rajagopalan and Tien (1976) reads:

$$\eta_G = N_G \left(\frac{a_s}{b} \right)^2 \qquad (1)$$

[Equation (23) in the above paper.] The dimensionless group, N_G, is the ratio of the net weight to the Stokes drag acting on the particle. Thus, the group N_G, which is equivalent to the product GaSt used by Tardos and Pfeffer (1980), measures the relative importance of the body forces acting on the particle over the convective influence of fluid motion. The variables a_s and b in Equation (1) above are the radii of the collector and the Happel cell, respectively. The appearance of b in Equation (1) makes η_G a function of the porosity, ϵ, of the packed bed.

The equation for gravitational collection efficiency as given in Tardos and Pfeffer (1980) reads:

$$E_G = \frac{GaSt}{1 + GaSt} = \frac{N_G}{1 + N_G} \qquad (2)$$

and is independent of the porosity ϵ, of the packed bed. Equation (2) reduces to the form:

$$E_G \simeq N_G \qquad (3)$$

for small values of this parameter. Comparing Equations (1) and (3) and assuming that the collection efficiencies η_G and E_G were defined equivalently led Tardos and Pfeffer (1980) to conclude that Equation (1) is incorrect. However it can be shown that Equation (1) is equivalent to Equation (2) because of a difference in the definition of the single-particle efficiencies.

One way is to define the efficiency as the ratio of the amount of particles colliding with the collector to the amount that passes through the projected area of the collector; this defines what may be called the collector efficiency, and leads to Equation (2) above. The second way of defining the efficiency is to normalize the amount of particles deposited by the total amount passing through the Happel cell. This may be called the bed efficiency, and leads to Equation (1). Thus:

$$\eta_{G,Bed} = \eta_{G,C} \cdot \left(\frac{a_s}{b}\right)^2 \qquad (4)$$

As stated on pp. 526–527 of Rajagopalan and Tien (1976), the latter definition has been used in that paper for two reasons. Firstly, it is consistent with the view that the Happel model is a mapping of the packed bed into the cell. Secondly, this definition guarantees, as stated on page 527 following Equation (17) in Rajagopalan and Tien (1976), that the maximum efficiency will never exceed unity.

We would like to emphasize at this point that the second definition (based on the cell diameter) is not appropriate for the case of deposition on *single* collectors. In this case this definition will lead to an efficiency of zero (i.e., $\eta_{Bed} \rightarrow 0$ as $\eta \rightarrow 1$) since the fractional deposition in an infinitely dilute "bed" would be expected to approach zero.

The definition of diffusional collection efficiency given in Equation (18) of Rajagopalan and Tien (1976) is however inconsistent with the interception and gravitational efficiencies, since Equation (18) is based on the collector efficiency and not on the bed efficiency. To be consistent, Equation (18) should read:

$$\eta_{Diff} = 4A_s^{1/3}N_{Pe}^{-2/3}\left(\frac{a_s}{b}\right)^2 \qquad (5)$$

Correspondingly, the last term of Equation (27) should also be multiplied by the factor $(a_s/b)^2$. These comments also apply to the equations presented in the recent review of deep bed filtration in Rajagopalan and Tien (1979).

We would like to make one additional remark regarding the additivity of the interception efficiency and the gravitational deposition efficiency. An analysis based on non-inertial motion of a particle of arbitrary radius and mass will show that the above two mechanisms do not act independently as shown, for example, in Tardos and Pfeffer (1980). However, the additive expressions given in Equations (25), (26), and (27) of Rajagopalan and Tien (1976) are valid, since they are interpolating equations obtained from numerical calculations.

LITERATURE CITED

Rajagopalan, R., and C. Tien, "Trajectory Analysis of Deep Bed Filtration with the Sphere-in-Cell Porous Media Model", AIChE J., **22**(3), 523 (1976).

Rajagopalan, R., and C. Tien, "Theory of Deep Bed Filtration, pp. 179–269 in Progress in Filtration and Separation", Vol. I, Wakeman, R. J. (Ed.), Elsevier, Amsterdam, 1979.

Tardos, G., and R. Pfeffer, "Interceptional and Gravitational Deposition of Inertialess Particles On a Single Sphere and In a Granular Bed", AIChE J., **26**(4), 698 (1980).

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